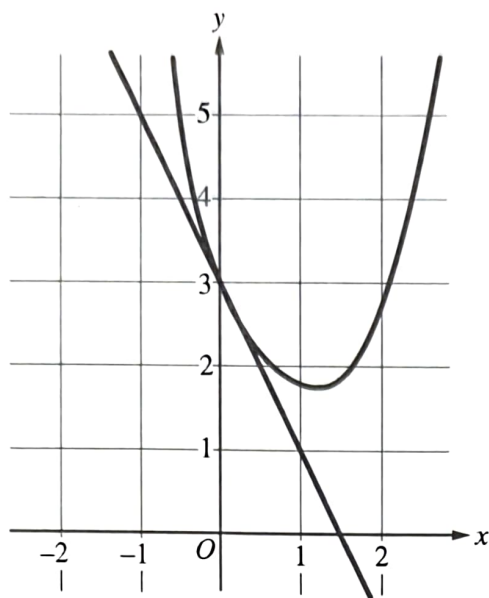


2019 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS



$n$	$f^{(n)}(0)$
2	3
3	$-\frac{23}{2}$
4	54

6. A function  $f$  has derivatives of all orders for all real numbers  $x$ . A portion of the graph of  $f$  is shown above, along with the line tangent to the graph of  $f$  at  $x = 0$ . Selected derivatives of  $f$  at  $x = 0$  are given in the table above.
- (a) Write the third-degree Taylor polynomial for  $f$  about  $x = 0$ .
- (b) Write the first three nonzero terms of the Maclaurin series for  $e^x$ . Write the second-degree Taylor polynomial for  $e^x f(x)$  about  $x = 0$ .
- (c) Let  $h$  be the function defined by  $h(x) = \int_0^x f(t) dt$ . Use the Taylor polynomial found in part (a) to find an approximation for  $h(1)$ .
- (d) It is known that the Maclaurin series for  $h$  converges to  $h(x)$  for all real numbers  $x$ . It is also known that the individual terms of the series for  $h(1)$  alternate in sign and decrease in absolute value to 0. Use the alternating series error bound to show that the approximation found in part (c) differs from  $h(1)$  by at most 0.45.

**STOP**  
**END OF EXAM**